A PROCESS FOR SYNTHESIZING BANDLIMITED CHAOTIC WAVEFORMS FOR DIGITAL SIGNAL TRANSMISSION

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Abstract

In the development of a chaotic oscillator technology to produce high-quality communication signals, the authors found a novel method for limiting the out-of-band spectral power from chaotic oscillators. This development is an important breakthrough that has allowed the authors to make a major step toward a commercially viable technology.

Introduction

In a practical wireless communication system, the signal transmitted from the antenna must be limited to a finite range of frequencies. Such signals are known as *bandlimited* signals; or, if the range of frequencies does not include dc-where the frequency equals 0 Hz—they are called *passband* signals. Typically, passband signals are bandlimited signals that occupy a small percentage of bandwidth about a center or carrier frequency. Multiple signals can then be transmitted by using passbands that do not overlap, then separating the signals at the receivers by filtering all but the passband of interest. This method of sending multiple signals, known as frequency-division multiplexing, is the basis for many multiple-user systems in use today. When many users can access a system on either a fixed- or flexible-frequency division plan, the method is known as frequency-division multiple access. Although some methods such as code division multiple access, or CDMA, do not rely upon frequency division for signal separation, the signals are still limited to a passband defined by the Federal Communications Commission (FCC) [1].

If the signals generated by chaotic systems are to be used in commercially viable systems, the transmitted signal must be bandlimited in some way. One obvious and often-used way to do this is to generate a signal that has considerable out-of-band spectral power, and then filter the signal to remove most of the power that lies outside of the desired frequency band. This method is, however, costly in many ways. Filters that effectively remove out-of-band energy, while passing the signal in-band with minimal distortion, are expensive and generally have a large footprint. The power outside of the band is also wasted, and must be dissipated as heat. This wasted power translates into increased transmitter power requirements and faster battery drain.

In this study, development efforts to produce a commercially viable chaotic oscillator technology yielded a particularly simple and effective way to limit the out-of-band radiation from a chaotic oscillator. This method caused the oscillator itself to produce a signal with bandwidth-constrained signal power and, thus, there was no need for a filter or waste of power. In doing so, the authors applied a principle that may be used for more general signal shaping or spectral shaping.

The idea behind this bandlimited–chaotic-oscillation (BCO) synthesis method was based on the segment-hopping method of oscillator control [2]. In segment hopping, a digital source produces an analog waveform that is used to guide the transmit oscillator. The guide signal is an analog copy of a signal that could be produced by the transmit oscillator itself, except that it follows a pre-defined symbol sequence that contains the digital information being transmitted. In this scheme, the transmit oscillator is acting as an amplifier for the guide signal, because the output of the transmitter can be much higher in power than the power drawn from the guide source.

Synthesis and Analysis

A prototypical chaotic oscillator used to produce signals useful for digital communication is the Lorenz system [3]. It was the oscillator first used by Hayes [4] to introduce the notion of controlling symbolic dynamics, a process to encode digital information in the oscillations of a chaotic system. The Lorenz system is described by a three-dimensional system of equations having the form:

$$\begin{aligned} \dot{x} &= \sigma y - \sigma x \\ \dot{y} &= \rho x - y - xz \\ \dot{z} &= xy - \beta z \end{aligned} \tag{1}$$

where *s*, *r*□ and *b*□ are parameters that Lorenz orignally set to 10, 28, and 8/3, respectively. The state-space attractor defined by these equations takes on a double-lobed structure, which lends itself nicely to a binary-symbol partition. Figure 1 is an example of a two-dimensional projection of the solutions of the Lorenz equations, showing state-variables *x* and *y*. Two Poincaré surfaces are placed in positions piercing the focal points and extending outward beyond the boundaries of attractor.

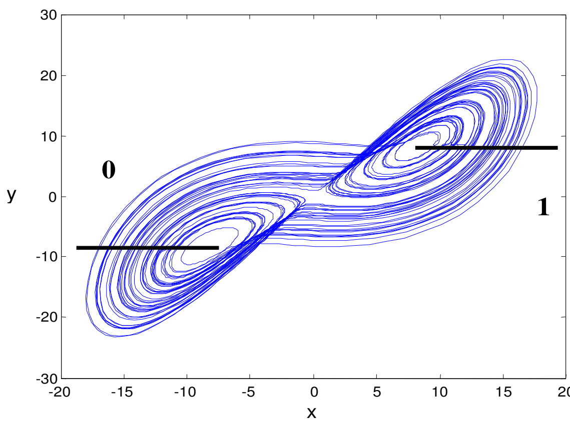

Figure 1. Two-dimensional projection of the solutions to the Lorenz equations showing state coordinates *x* and *y* and the binary partitioning of the state-space

The time-varying state-variable, $x(t)$, is a bipolar waveform well-suited for baseband transmission of digital information. Figure 2 is an example of the waveform in its generalized time coordinates, and Figure 3 is a plot of the frequency content of this signal with respect to its average cycle frequency. The average cycle frequency is the reciprocal of the average cycle time. The cycle time is defined as the time it takes a point on the attractor to travel from one Poincaré surface to the other, or back to itself. The average cycle time is the mean time, in seconds, calculated by integrating the system with a fixed integration step and collecting thousands of surface crossings. T_{avg} was found to be about 1.7 seconds, with a variance of about 1.8 seconds.

Figure 4 is a block diagram of the implementation of a bandlimited chaotic signal source. A binary sequence is fed into a discrete-time, bandlimited, segment-hopping source. This source will be described in the next section. The output signal, $y[n;t]$, is a sampled waveform used as a guide signal to synchronize a continuous-time Lorenz oscillator to it. A single state-variable synchronization method is used to lock the oscillator to the dynamics and, thus, to the embedded digital sequence of the guide signal.

Since Pecora and Carroll first introduced the concept of synchronization of chaotic oscillator circuits [5], many methods have been developed and realized.

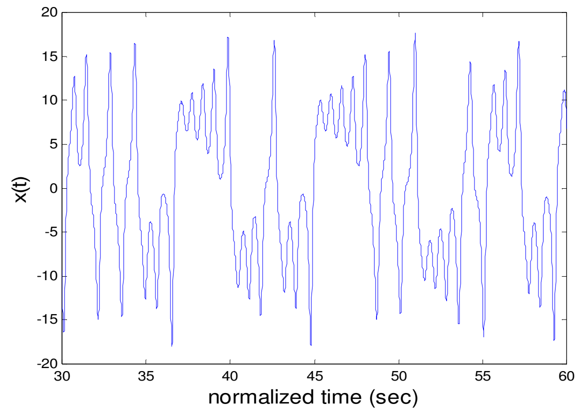

Figure 2. Time-dependent solution of the Lorenz equations for state coordinate $x(t)$

The following mathematical model works extremely well:

$$\begin{aligned} \dot{x} &= \sigma y - \sigma x \\ \dot{y} &= \rho x - y - xz + \left(y[n;t] - y\right)R \\ \dot{z} &= xy - \beta z \end{aligned} \tag{2}$$

Here, R is a coupling factor. This method works well from the standpoint of simplicity and ultimately from a circuit-realization perspective. There have been circuit realizations proposed for the Lorenz equations [6]. In such circuits this coupling method is simply a resistive feed of a voltage across, or current into, a single arm.

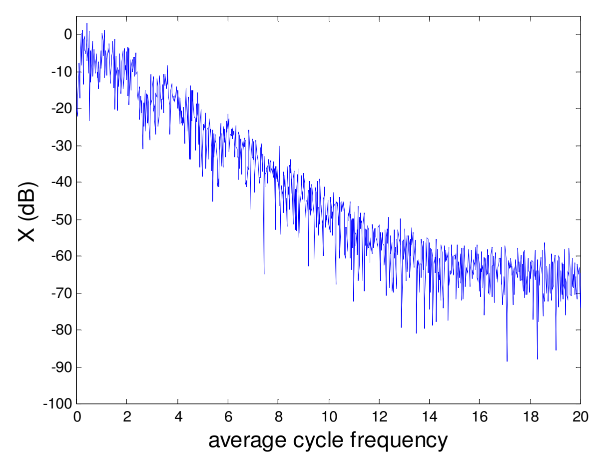

Figure 3. Frequency content of the state coordinate $x(t)$ with respect to the average cycle frequency

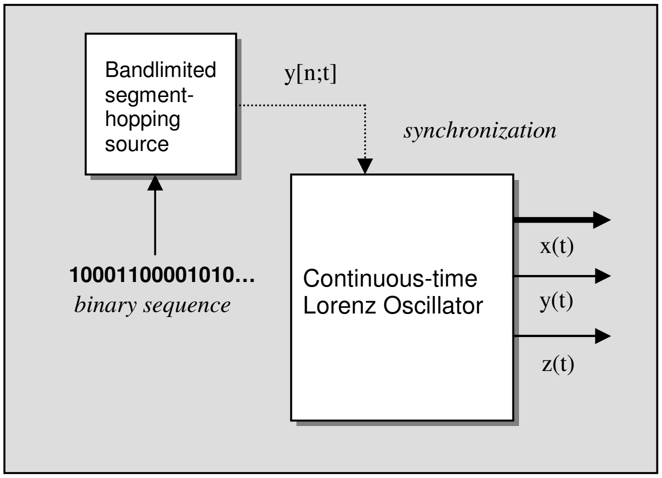

Figure 4. Block diagram of the implementation of a bandlimited digital signal synthesizer

Segment Hopping

Refer to the synthesis of the guide signal, $y[n;t]$. Two new concepts are introduced here.

1. Continuous, digitally-encoded chaos waveforms can be generated by piecing together the proper signal segments.
2. These segments can be bandlimited prior to storage, and can impress their characteristics upon a continuous-time oscillator via synchronization.

The sequencing of stored segments in memory in some pre-defined order is commonly used in arbitrary waveform synthesis. What is new and not obvious about segment hopping is that an arbitrary controlled *trajectory* of a deterministic dynamic system can be generated, even though the system produces continuous and non-repeating trajectories in state space under the action of deterministic differential equations. Furthermore, this signal can be made to carry an arbitrary sequence of digital symbols representing encoded data. Thus, the difference between this method and arbitrary waveform synthesis is that this method allows for the production of a signal carrying arbitrary data that appears to have been produced by the action of differential equations. This is achieved by putting out segments that follow the desired symbol sequence, while satisfying the grammar of the oscillator. The theoretical basis for this method of signal synthesis is the idea from ergodic theory that chaos can be approximated to an arbitrary degree of accuracy by completely deterministic mappings in state space intermediated by completely random choices [7].

For example, in an 8-bit encoding there are 256 signal pieces, or segments, that can be put together to form any desired binary-symbol sequence. The segments are assigned numbers from 0 to 255 according to the bit sequence they initiate. In a physical implementation, a segment-hopping system can be stored in a static memory device such as a ROM or EPROM and clocked out according to the input bit-sequence. A more detailed description of the segment-hopping process will be published shortly. In the following section, the authors consider a complete computer model of the BCO synthesis method.

Computer Model Results

The first, important step was to determine the amount of bandlimiting that can be achieved. Any amount of filtering will cause some distortion to the waveforms. It was then important that the binary encoding remain preserved, and that the guide signal remain capable of synchronizing the continuous-time oscillator. Given the determination of the average cycle time, T_{avg}, as stated earlier, a smooth, low-pass filter was applied to the Lorenz oscillations having a cutoff-frequency of twice the average bit rate. With the filtering applied, segments were stored that source the appropriate 8-bit symbol sequences. Figures 5 and 6 show the filter characteristics and the response of the frequency spectra and the resultant attractor.

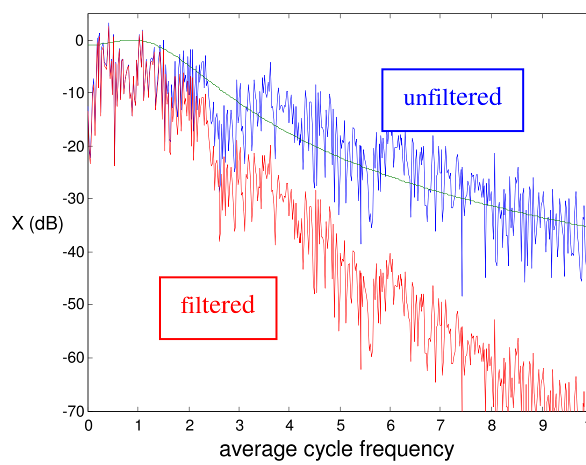

Figure 5. The result of the low-pass filtering of the state variable of *x*(*t*)

The filtering was applied to all three state coordinates in order to produce a new attractor. It was with this new attractor that the symbolic dynamics of the system were determined and the associated segments produced.

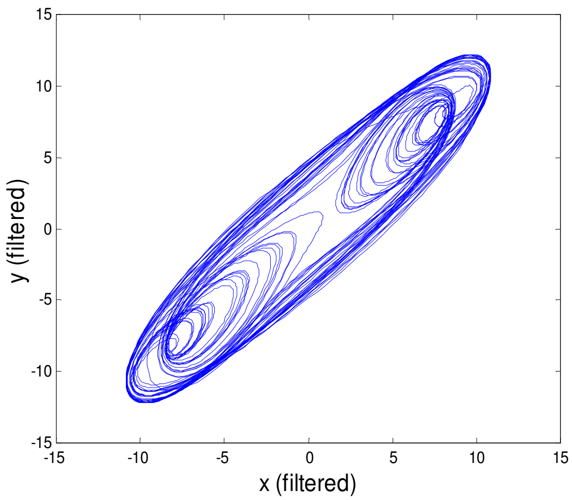

Figure 6. Two-dimensional projection of a filtered Lorenz oscillation using the low-pass filter characteristic shown by its effect on the spectrum in Figure 5

Even though only one state-coordinate was used as a guide signal, namely *y*, the entire attractor had to be transformed in order to synthesize the segments needed for segment-hopping control.

In the following example, the binary sequence was encoded and transmitted as

B = 11001101100010101111000110101100011

Figure 7 shows what the desired output waveform, $x(t)$, would be for a typical Lorenz oscillator. Given the bandlimited segments, the guide signal, $y[n]$, could be synthesized. Figure 8 is a plot of the frequency spectrum of the original, unfiltered transmit signal, $x(t)$, the result of which was filtered using the low-pass filter described earlier, and the frequency spectrum of the new transmit signal from the BCO synthesis method.

Note the dramatic reduction in frequency content. Particularly, there was nearly a 20 dB reduction at 3.5 times the cycle frequency and a 30 dB reduction at 6 times the cycle frequency. Earlier it was stated that it is important that binary encoding remain preserved and that the synthesized BCO guide signal be capable of synchronizing a Lorenz oscillator to it. This is demonstrated in Figures 9 and 10.

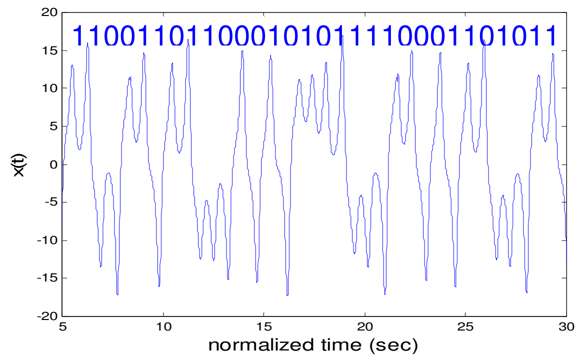

Figure 7. Typical Lorenz oscillation producing the binary sequence:
B = 11001101100010101111000110101100011

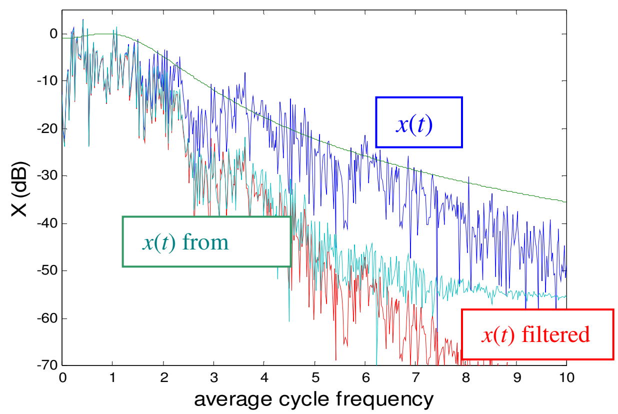

Figure 8. Frequency spectra of $x(t)$ from the Lorenz equations, a filtered version, and the output of a BCO system

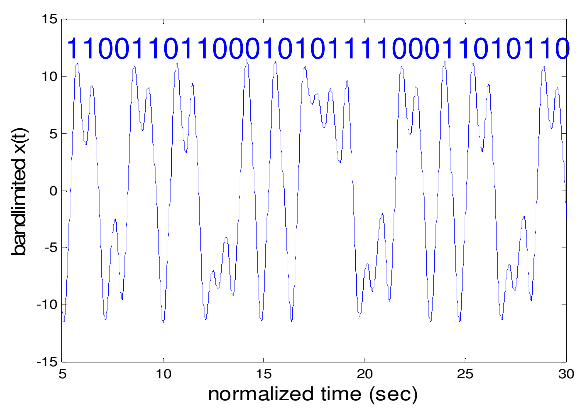

Figure 9. BCO-synthesized transmit signal with the binary encoding preserved

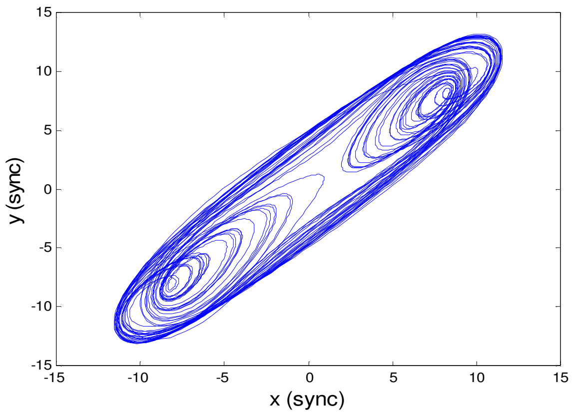

Figure 10. BCO-synthesized attractor projection

What was remarkable was how "flexible" the Lorenz oscillator can be. This was an important realization not only for this work but for synthesizing signals even more compatible with traditional communication signal formats and standards. It was demonstrated in the lab in prototypes that physical chaotic oscillators can be made to produce signals that have constant-timing intervals between Poincaré surface crossings by simply synchronizing them to an artificially-synthesized waveform having those properties. This timing regularization is of utmost importance for use in commercially-viable communication systems because it makes accurate clock-timing recovery possible. Combined with bandwith compression, as outlined here, this solved some of the most critical technological problems for using chaotic oscillators in commercial systems.

Conclusion

What was described here in a simple example was an enabling technology. In order to use chaotic systems and processes in commercial digital communication systems, the waveforms must be restricted in bandwidth in a controllable way. The focus must shift to the *design* of waveforms and sources, and away from the use of existing, easily-constructed oscillators.

The BCO synthesis technique in this study demonstrated a simple, efficient and effective way to generate baseband signals for wireless communication. Some chaotic oscillators, such as the Colpitt's circuit [8] and the double-scroll oscillator [9], produce signals with very different characteristics. However, this technique is a general procedure applicable to a variety of chaotic oscillators.

Another area of technology development that the authors are focusing on is in the development of oscillators that are ideally suited for given communication channels. Although the Lorenz system has excellent properties for binary baseband digital signaling in white noise, there may be oscillations better suited and which can produced with more efficient circuitry.

Since Ott, Grebogi, and Yorke's initial formalism for controlling chaotic processes using small perturbations [10], there has been a consistent march toward development of technological applications. Arguably, the application to communications technology has held the most promise and has been the source of the most activity. Beyond simple applications is the realm of *commercially viable* applications that have characteristics that make chaotic dynamics technology attractive to the fast-moving world of the telecommunications industry. Some of the improvement areas that make chaotic dynamics technology attractive are increases in efficiency, reduction of system complexity, increases in transmission ranges and digital transmission data rates.

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Biography

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